

# Improved Ratio Estimator of the Population Mean Using Power Transformation in Simple Random Sampling

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## Abstract

This paper proposes an improved ratio estimator of the population mean of study variable using the power transformation approach under simple random sampling without replacement (SRSWOR). The bias and mean square error of the proposed estimator up to the first order of approximation were derived. The biases, mean square errors (MSEs) and percent relative efficiencies (PREs) were computed using real-life data from three populations. The theoretical and empirical evaluations demonstrate that the proposed estimator outperforms existing ratio estimators considered, providing more accurate and reliable estimates. The results of this study contribute to the development of the methods in survey sampling.

**Keywords:** Ratio Estimator, Mean Square Error, Efficiency, Auxiliary Information, Power Transformation.

## Introduction

In survey sampling, accurate estimation of population parameters is essential for informed decision-making. Ratio-type estimators have been used widely to estimate population parameters by integrating auxiliary information to improve estimation efficiency as pioneered [1,2].

Numerous efforts have been made to improve on the precision of the ratio estimator proposed through the search for the estimator with higher efficiency that has minimum variance or mean square error [1]. This is achieved through the use of linear combinations of the auxiliary variable, use of unknown weights and power transformation approach and other researchers have modified the ratio estimator in search for efficiency gain [3-7].

The power transformation method is a widely used technique in ratio estimation to improve the accuracy of the population mean estimates and used this technique by incorporating an unknown weight to improve on the estimator proposed by [8,9].

Thus, this paper aims at incorporating the correlation coefficient and the sample size of the population to develop a new estimator that is both effective and efficient in improving the accuracy of estimating the population mean in simple random sampling without replacement through the use of power transformation.

## Sampling Procedure and Notations

Let  $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_N\}$  be a finite population of size  $N$  and let

$(y_i, x_i)$  be the value of the study variable  $Y$  and the auxiliary variable  $X$  on  $i^{th}$  unit  $\Omega_i, i = 1, 2, \dots, N$ . Let  $\bar{Y}$  and  $\bar{X}$  be population means of the study variable  $Y$  and the auxiliary variable  $X$  respectively. Let a sample of size  $(n)$  be drawn by simple random sampling without replacement (SRSWOR) based on which the means  $(\bar{x})$  and  $(\bar{y})$  for the auxiliary variable  $(X)$  and the study variable  $(Y)$  are obtained. We assume that the population mean  $\bar{X}$  and the population variance  $S_x^2$  of the auxiliary variable are known. Thus, the following notations are defined

$\bar{y} = n^{-1} \sum_{i=1}^n y_i$  : Sample mean of the study variable,

$\bar{x} = n^{-1} \sum_{i=1}^n x_i$  : Sample mean of the auxiliary variable,

$\bar{Y} = N^{-1} \sum_{i=1}^N Y_i$  : Population mean of the study variable,

$\bar{X} = N^{-1} \sum_{i=1}^N X_i$  : Population mean of the auxiliary variable,

$C_x = S_x \bar{X}^{-1}$  : Coefficients of variation of the auxiliary variables,

$C_y = S_y \bar{Y}^{-1}$  : Coefficients of variation of the study variable,

$\bar{X} = N^{-1} : \text{Population mean of the auxiliary variable,}$

$C_x = S_x \bar{X}^{-1} : \text{Coefficients of variation of the auxiliary variables,}$

$C_y = S_y \bar{Y}^{-1} : \text{Coefficients of variation of the study variable,}$

$\rho = S_{xy} (S_y S_x)^{-1} : \text{Population correlation coefficient between the auxiliary and the study variables}$

$S_x^2 = (N-1)^{-1} \sum_{i=1}^N (X_i - \bar{X})^2 : \text{Population variance of the auxiliary variable,}$

$S_y^2 = (N-1)^{-1} \sum_{i=1}^N (Y_i - \bar{Y})^2 : \text{Population variance of the study variable,}$

$S_{xy} = (N-1)^{-1} \sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y}) : \text{Covariance between the study and auxiliary variables,}$

$\beta_1 = [(N-1)(N-2)S_x^2]^{-1} N \sum_{i=1}^N (X_i - \bar{X})^3 : \text{Coefficient of skewness of the auxiliary variable,}$

$\beta_2 = [(N-1)(N-2)(N-3)S_x^4]^{-1} N(N+1) \sum_{i=1}^N (X_i - \bar{X})^4 - [(N-2)(N-3)]^{-1} 3(N-1)^2$   
:Coefficient of kurtosis of the auxiliary variable, : the median of the auxiliary variable.

## Review of Existing Related Estimators

The sample mean estimator of the population mean and its variance are given as

$$t_0 = \frac{\sum_{i=1}^n y_i}{n} \quad (1)$$

$$Var(t_0) = \bar{Y}^2 \eta C_y^2 \quad (2)$$

The use of auxiliary information ( $X$ ) and suggested the classical ratio estimator for estimating population mean ( $\bar{Y}$ ) of the study variable ( $Y$ ) [1]. The suggested estimator, its bias and mean square error are given, respectively as

$$t_r = \frac{\bar{y}}{\bar{x}} \bar{X} \quad (3)$$

$$Bias(t_r) = \bar{Y} \eta [C_x^2 - \rho C_y C_x] \quad (4)$$

$$MSE(t_r) = \bar{Y}^2 \eta [C_y^2 + C_x^2 - 2\rho C_y C_x] \quad (5)$$

Proposed ratio type estimator of the population by considering the use of known value of coefficient of variation ( $C_x$ ) of the auxiliary variable [10]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_1 = \bar{y} \left( \frac{\bar{X} + C_x}{\bar{x} + C_x} \right) \quad (6)$$

$$Bias(t_1) = \bar{Y} \eta [\phi_1^2 C_x^2 - \phi_1 \rho C_y C_x] \quad (7)$$

$$MSE(t_1) = \bar{Y}^2 \eta [C_y^2 + \phi_1^2 C_x^2 - 2\phi_1 \rho C_y C_x] \quad (8)$$

$$\text{Where, } \phi_1 = \frac{\bar{X}}{\bar{X} + C_x}$$

Proposed ratio type estimator of the population by imposing coefficient of variation ( $C_x$ ) and coefficient of kurtosis ( $\beta_2$ ) of auxiliary variable on the work [5,10]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_2 = \bar{y} \left( \frac{\bar{X} C_x + \beta_2}{\bar{x} C_x + \beta_2} \right) \quad (9)$$

$$Bias(t_2) = \bar{Y} \eta [\phi_2^2 C_x^2 - \phi_2 \rho C_y C_x] \quad (10)$$

$$MSE(t_2) = \bar{Y}^2 \eta [C_y^2 + \phi_2^2 C_x^2 - 2\phi_2 \rho C_y C_x] \quad (11)$$

$$\text{Where, } \phi_2 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2}$$

A new estimator of the population mean by incorporating coefficient of correlation ( $\rho$ ) of the auxiliary variable in the work [1,11]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_3 = \bar{y} \left( \frac{\bar{X} + \rho}{\bar{x} + \rho} \right) \quad (12)$$

$$Bias(t_3) = \bar{Y} \eta [\phi_3^2 C_x^2 - \phi_3 \rho C_y C_x] \quad (13)$$

$$MSE(t_3) = \bar{Y}^2 \eta [C_y^2 + \phi_3^2 C_x^2 - 2\phi_3 \rho C_y C_x] \quad (14)$$

$$\text{Where, } \phi_3 = \frac{\bar{X}}{\bar{X} + \rho}$$

A new ratio estimator of population mean by incorporating coefficient of kurtosis ( $\beta_2$ ) of the auxiliary variable [11,12]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_4 = \bar{y} \left( \frac{\bar{X} + \beta_2}{\bar{x} + \beta_2} \right) \quad (15)$$

$$Bias(t_4) = \bar{Y} \eta [\phi_4^2 C_x^2 - \phi_4 \rho C_y C_x] \quad (16)$$

$$MSE(t_4) = \bar{Y}^2 \eta [C_y^2 + \phi_4^2 C_x^2 - 2\phi_4 \rho C_y C_x] \quad (17)$$

$$\text{Where, } \phi_4 = \frac{\bar{X}}{\bar{X} + \beta_2}$$

A new ratio type estimator of population mean by incorporating known value of coefficient of skewness ( $\beta_1$ ) of the auxiliary variable [6]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_5 = \bar{y} \left( \frac{\bar{X} + \beta_1}{\bar{x} + \beta_1} \right) \quad (18)$$

$$Bias(t_5) = \bar{Y} \eta \left[ \varphi_5^2 C_x^2 - \varphi_5 \rho C_y C_x \right] \quad (19)$$

$$MSE(t_5) = \bar{Y}^2 \eta \left[ C_y^2 + \varphi_5^2 C_x^2 - 2\varphi_5 \rho C_y C_x \right] \quad (20)$$

$$\text{where, } \varphi_5 = \frac{\bar{X}}{\bar{X} + \beta_1}$$

A new ratio type estimator of population mean by incorporating known value of median ( $M_x$ ) of the auxiliary variable [13]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_6 = \bar{y} \left( \frac{\bar{X} + M_x}{\bar{x} + M_x} \right) \quad (21)$$

$$Bias(t_6) = \bar{Y} \eta \left[ \varphi_6^2 C_x^2 - \varphi_6 \rho C_y C_x \right] \quad (22)$$

$$MSE(t_6) = \bar{Y}^2 \eta \left[ C_y^2 + \varphi_6^2 C_x^2 - 2\varphi_6 \rho C_y C_x \right] \quad (23)$$

$$\text{where, } \varphi_6 = \frac{\bar{X}}{\bar{X} + M_x}$$

The estimator of population mean, introduced ample size of the sample of population [1,8]. The proposed estimator, its bias and mean square error are given, respectively as

$$t_7 = \bar{y} \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \quad (24)$$

$$Bias(t_7) = \bar{Y} \eta \left[ \varphi_7^2 C_x^2 - \varphi_7 \rho C_y C_x \right] \quad (25)$$

$$MSE(t_7) = \bar{Y}^2 \eta \left[ C_y^2 + \varphi_7^2 C_x^2 - 2\varphi_7 \rho C_y C_x \right] \quad (26)$$

$$\text{where, } \varphi_7 = \frac{\bar{X}}{\bar{X} + n}$$

The estimator is incorporating coefficient of correlation ( $\rho$ ) The proposed estimator, its bias and mean square error are given, respectively as [8,14]

$$t_8 = \bar{y} \left( \frac{\bar{X} \rho + n}{\bar{x} \rho + n} \right) \quad (27)$$

$$Bias(t_8) = \bar{Y} \eta \left[ \varphi_8^2 C_x^2 - \varphi_8 \rho C_y C_x \right] \quad (28)$$

$$MSE(t_8) = \bar{Y}^2 \eta \left[ C_y^2 + \varphi_8^2 C_x^2 - 2\varphi_8 \rho C_y C_x \right] \quad (29)$$

$$\text{where, } \varphi_8 = \frac{\bar{X} \rho}{\bar{X} \rho + n}$$

The use of unknown weight and defined a new ratio type estimator of population mean [7]. The proposed estimator, its bias and mean square error respectively are given as

$$t_9 = \bar{y} \alpha \left( \frac{\bar{X} + n}{\bar{x} + n} \right) \quad (30)$$

$$Bias(t_9) = \bar{Y} \left[ (\alpha - 1) + \eta \varphi_9^2 C_x^2 - \eta \varphi_9 \rho C_y C_x \right] \quad (31)$$

$$MSE(t_9) = \bar{Y}^2 \left[ 1 - \frac{(1 + \eta \varphi_9^2 C_x^2 - \eta \varphi_9 \rho C_y C_x)^2}{1 + \eta C_y^2 + 3\eta \varphi_9^2 C_x^2 - 4\eta \varphi_9 \rho C_y C_x} \right] \quad (32)$$

$$\text{where, } \varphi_9 = \frac{\bar{X}}{\bar{X} + n} \text{ and } \alpha = \frac{1 + \eta \varphi_9^2 C_x^2 - \eta \varphi_9 \rho C_y C_x}{1 + \eta C_y^2 + 3\eta \varphi_9^2 C_x^2 - 4\eta \varphi_9 \rho C_y C_x}$$

## Methodology

### Proposed Estimator

In this section, a new ratio type estimator of the population mean is proposed by incorporating known value of the coefficient of correlation of the auxiliary variable in order to improve on the precisions of ratio type estimators [8,14]. The proposed estimator is given as

$$t^{NT} = \bar{y} \left( \frac{\bar{X} \rho + n}{\bar{x} \rho + n} \right)^\gamma \quad (33)$$

The asymptotic properties of the proposed estimator are obtained by defining the following terms

$$\text{Let } \zeta_y = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \text{ and } \zeta_x = \frac{\bar{x} - \bar{X}}{\bar{X}}, \text{ such that } \bar{y} = \bar{Y} [1 + \zeta_y]$$

$$\text{and } \bar{x} = \bar{X} [1 + \zeta_x]$$

$$\left. \begin{aligned} E(\zeta_y) &= E(\zeta_x) = 0 \\ E(\zeta_y^2) &= \eta C_y^2, E(\zeta_x^2) = \eta C_x^2, E(\zeta_x \zeta_y) = \eta \rho C_y C_x \end{aligned} \right\} \quad (34)$$

$$\text{where, } \eta = \left( \frac{1}{n} - \frac{1}{N} \right)$$

Expressing (33) in terms of  $\zeta$  and  $\zeta_x$  gives

$$t^{NT} = \bar{Y} [1 + \zeta_y] \left[ \frac{\bar{X} \rho + n}{\bar{X} \rho (1 + \zeta_x) + n} \right]^\gamma \quad (35)$$

Factorizing the second component of the RHS of (35) gives

$$t^{NT} = \bar{Y} [1 + \zeta_y] \left[ \frac{\bar{X} \rho + n}{\bar{X} \rho + n \left( 1 + \frac{\bar{X} \rho \zeta_x}{\bar{X} \rho + n} \right)} \right]^\gamma \quad (36)$$

$$t^{NT} = \bar{Y} [1 + \zeta_y] \left[ \frac{1}{(1 + \phi \zeta_x)} \right]^\gamma \quad (37)$$

where,  $\varphi = \frac{\bar{X}\rho}{\bar{X}\rho + n}$

$$t^{NT} = \bar{Y} [1 + \zeta_y] [1 + \varphi \zeta_x]^{-\gamma} \quad (38)$$

If it is assumed that  $|\varphi \zeta_x| < 1$ , then the expression  $[1 + \varphi \zeta_x]^{-\gamma}$  can be expanded to convergent infinite series using binomial expansion, and approximating to the first order, by neglecting terms of greater than two gives

$$t^{NT} = \bar{Y} [1 + \zeta_y] \left[ 1 - \gamma \varphi \zeta_x + \frac{\gamma(\gamma-1)}{2} \varphi^2 \zeta_x^2 \right] \quad (39)$$

Multiplying out the brackets gives

$$t^{NT} = \bar{Y} + \bar{Y} \left[ \zeta_y - \gamma \varphi \zeta_x + \frac{\gamma(\gamma-1)}{2} \varphi^2 \zeta_x^2 - \gamma \varphi \zeta_x \zeta_y \right] \quad (40)$$

Subtracting and taking expectation to both sides of (40)

$$E[t^{NT} - \bar{Y}] = E \left[ \bar{Y} \left( \zeta_y - \gamma \varphi \zeta_x + \frac{\gamma(\gamma-1)}{2} \varphi^2 \zeta_x^2 - \gamma \varphi \zeta_x \zeta_y \right) \right] \quad (41)$$

Applying results of (34), then, the bias of the estimator is obtained as

$$\text{Bias}(t^{NT}) = \bar{Y} \eta \left[ \frac{\gamma(\gamma-1)}{2} \varphi^2 C_x^2 - \gamma \varphi \rho C_y C_x \right] \quad (42)$$

Also, subtracting, taking expectation and squaring both sides of (41) gives

$$E[t^{NT} - \bar{Y}]^2 = E \left[ \bar{Y} \left( \zeta_y - \gamma \varphi \zeta_x + \frac{\gamma(\gamma-1)}{2} \varphi^2 \zeta_x^2 - \gamma \varphi \zeta_x \zeta_y \right) \right]^2 \quad (43)$$

Applying results of (34), then, the mean square error (MSE) of the estimator  $t^{NT}$  is obtained as

$$MSE(t^{NT}) = \bar{Y}^2 \eta \left[ C_y^2 + \gamma^2 \varphi^2 C_x^2 - 2\gamma \varphi \rho C_y C_x \right] \quad (44)$$

To obtain the optimum value of  $\gamma$ , we differentiate (44) partially with respect to  $\gamma$  and equate to zero as

$$\frac{\partial MSE(t^{NT})}{\partial \gamma} = \bar{Y}^2 \eta \left[ 2\gamma \varphi^2 C_x^2 - 2\varphi \rho C_y C_x \right] = 0$$

$$2\gamma \varphi^2 C_x^2 = 2\varphi \rho C_y C_x$$

$$\gamma^{opt} = \frac{\rho C_y}{\varphi C_x} \quad (45)$$

By substituting the value of (45) in (44), the minimum mean square error is obtained as

$$MSE(t^{NT})_{\min} = \bar{Y}^2 \eta \left[ C_y^2 + \left( \frac{\rho C_y}{\varphi C_x} \right)^2 \varphi^2 C_x^2 - 2 \left( \frac{\rho C_y}{\varphi C_x} \right) \varphi \rho C_y C_x \right] \quad (46)$$

Simplifying (46) further, the minimum mean square error is obtained as

$$MSE(t^{NT})_{\min} = \bar{Y}^2 \eta C_y^2 [1 - \rho^2] \quad (47)$$

### Theoretical Efficiency Comparisons

Here, the efficiency of the proposed estimator is compared with some of the aforementioned estimators in the literature. Hence, the conditions under which the proposed estimator is more efficient are given as

i. The proposed estimator  $t^{NT}$  is more efficient than the estimator of the sample mean, if and only if

$$MSE(t^{NT}) < Var(t_0) \quad (48)$$

$$\begin{aligned} \bar{Y}^2 \eta C_y^2 [1 - \rho^2] &< \bar{Y}^2 \eta C_y^2 \\ 1 - \rho^2 &< 1 \end{aligned} \quad (49)$$

ii. The proposed estimator  $t^{NT}$  is more efficient than the usual estimator defined by Cochran (1940) if and only if

$$MSE(t^{NT}) < MSE(t_r) \quad (50)$$

$$\bar{Y}^2 \eta C_y^2 [1 - \rho^2] < \bar{Y}^2 \eta [C_y^2 + C_x^2 - 2\rho C_y C_x]$$

$$C_y^2 [1 - \rho^2] < C_y^2 + C_x^2 - 2\rho C_y C_x$$

$$C_y^2 [(1 - \rho^2) - 1] < C_x^2 - 2\rho C_y C_x \quad (51)$$

iii. The proposed estimator  $t^{NT}$  is more efficient than the other ratio type estimators aforementioned in the reviewed literature if and only if

$$MSE(t^{NT}) < MSE(t_i), i = 1, 2, \dots, 9 \quad (52)$$

$$\bar{Y}^2 \eta C_y^2 [1 - \rho^2] < \bar{Y}^2 \eta [C_y^2 + \varphi_i^2 C_x^2 - 2\varphi_i \rho C_y C_x]$$

$$C_y^2 [1 - \rho^2] < C_y^2 + \varphi_i^2 C_x^2 - 2\varphi_i \rho C_y C_x$$

$$C_y^2 [(1 - \rho^2) - 1] < \varphi_i^2 C_x^2 - 2\varphi_i \rho C_y C_x \quad (53)$$

### Percentage Relative Efficiency (PRE)

The percent relative efficiency (PRE) is used as the performance measure of estimator's efficiency gain. The PRE the proposed estimator  $t^{NT}$  and existing estimators  $t_i$  with respect to the usual ratio estimator  $t_r$  is given by

$$PRE = \frac{MSE(t_r)}{MSE(t^{NT})} \times 100 \text{ and } PRE = \frac{MSE(t_r)}{MSE(t_i)} \times 100, i = 1, 2, \dots, 9 \quad (54)$$

## Empirical Study and Results Discussion

In order to access or examine the performance of the proposed estimator and the existing related ratio estimators of the population mean using auxiliary variable, four real-life populations data sets were considered. The descriptive statistics for the real-life data sets are given in Table 1.

**Table 1: Population Descriptive Statistics for the Real Data Sets.**

| Parameters | Population I | Population II | Population III |
|------------|--------------|---------------|----------------|
| $N$        | 80           | 40            | 40             |
| $n$        | 20           | 8             | 8              |
| $\bar{Y}$  | 51.8246      | 50.7858       | 50.7858        |
| $\bar{X}$  | 11.2646      | 2.3033        | 9.4543         |
| $\rho$     | 0.9413       | 0.8006        | 0.8349         |
| $C_y$      | 0.3542       | 0.3295        | 0.3295         |
| $C_x$      | 0.7507       | 0.8406        | 0.6756         |
| $\beta_1$  | 1.0500       | 0.9740        | 0.8799         |
| $\beta_2$  | -0.0634      | -0.5344       | -0.4622        |
| $M_x$      | 7.5750       | 1.2500        | 7.0700         |

Population I and II [16].

Population III [15].

**Table 2: Bias Estimates of the Proposed and Existing Estimators.**

| Estimators          | Population I | Population II | Population III |
|---------------------|--------------|---------------|----------------|
| $t_r$               | 0.6088       | 2.4624        | 1.3742         |
| $t_1$               | 0.0506       | 0.2760        | 0.2573         |
| $t_2$               | 0.1134       | 3.7353        | 0.6588         |
| $t_3$               | 0.0350       | 0.3047        | 0.2225         |
| $t_4$               | 0.1156       | 3.1516        | 0.5777         |
| $t_5$               | 0.1170       | 0.1896        | 0.2131         |
| $t_6$               | -0.1901      | 0.0479        | -0.3213        |
| $t_7$               | -0.2083      | -0.3242       | -0.3424        |
| $t_8$               | -0.0372      | -0.0848       | 0.1015         |
| $t_9$               | -0.0355      | -0.2267       | -0.2062        |
| $t^{NT}$ (Proposed) | -0.1920      | -0.3642       | -0.4265        |

**Table 3: Mean Square Error of the Proposed and Existing Estimators.**

| Estimators          | Population I | Population II | Population III |
|---------------------|--------------|---------------|----------------|
| $t_r$               | 18.9793      | 95.8641       | 49.8536        |
| $t_1$               | 15.2581      | 42.0198       | 41.0685        |
| $t_2$               | 18.5128      | 217.7034      | 61.4577        |
| $t_3$               | 14.4503      | 43.47728      | 39.3031        |
| $t_4$               | 18.6279      | 188.0582      | 57.3389        |
| $t_5$               | 18.7015      | 37.6296       | 38.8230        |
| $t_6$               | 2.7825       | 30.4328       | 11.6863        |
| $t_7$               | 1.8289       | 11.5391       | 10.6117        |
| $t_8$               | 1.9700       | 12.9992       | 9.42415        |
| $t_9$               | 1.8389       | 11.5152       | 10.4705        |
| $t^{NT}$ (Proposed) | 1.4400       | 10.0540       | 8.48310        |

**Table 4: PRE of the Proposed and Existing Estimators.**

| Estimators          | Population I | Population II | Population III |
|---------------------|--------------|---------------|----------------|
| $t_r$               | 100          | 100           | 100            |
| $t_1$               | 124.3880     | 228.1403      | 121.3913       |
| $t_2$               | 102.5199     | 44.0343       | 81.1186        |
| $t_3$               | 131.3419     | 220.4924      | 126.8439       |
| $t_4$               | 101.8864     | 50.9758       | 86.9455        |
| $t_5$               | 101.4854     | 254.7572      | 128.4125       |
| $t_6$               | 682.0952     | 315.0026      | 426.5987       |
| $t_7$               | 1037.744     | 832.5027      | 469.7984       |
| $t_8$               | 963.416      | 737.4615      | 528.8998       |
| $t_9$               | 1032.100     | 832.5030      | 476.1330       |
| $t^{NT}$ (Proposed) | 1318.011     | 953.4940      | 587.6810       |

## Results Discussion

This study introduced a new ratio estimator for population mean estimation. We evaluated its performance against traditional ratio estimator and other existing estimators using three real datasets. The results, presented in Tables 3 and 4, show that the proposed estimator outperforms the others, exhibiting the lowest mean square error (MSE) and highest percentage relative efficiency (PRE). This indicates that our estimator provides more accurate and reliable estimates, with less dispersion from the true population mean.

## Conclusion

In conclusion, this study proposes a new ratio-type estimator for simple random sampling without replacement, integrating auxiliary information and a power transformation with unknown weight to develop the estimator. The properties (Bias and MSE) of the proposed estimator have been derived up to first order approximation using Taylor's series expansion method and its performance evaluated using three real data sets. The

results in Table 4 show that the proposed estimator outperforms other existing ones considered in this study. Consequently, the proposed estimator is recommended for practical use in estimating population mean of any variable of interest [15,16].

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**Declaration of Competing Interest:** The authors declare that they have no known competing financial interest

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